Indian Education Division
Montana Office of Public Instruction
opi.mt.gov

Mathematics – Statistics and Probability
Grades 9-12
Montana Native American Populations

Overview

Using population figures for Montana American Indians from 2000-2006, students will examine trends on Montana reservations and across the state. Students will compare American Indian populations with total populations and look for patterns on reservations (including surrounding areas) and generally across the state. Students will make predictions of future populations based on their assessment of population trends between 2000 and 2006. This learning activity is designed to be used with Algebra 2, Math III, or more advanced mathematics courses.

Timeframe: Two 50-minute class periods

Math Domains: Statistics and Probability; Interpreting Categorical and Quantitative Data; Linear, Quadratic, and Exponential Models

Strategies & Practices

Mathematical Practice (for student focus):

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations**.**
3. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. **Model with mathematics.** Mathematically proficient students … are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions.
5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.
6. **Attend to Precision.** Mathematically proficient students try to communicate precisely to others.

Standards

Math Clusters for Interpreting Categorical and Quantitative Data: Summarize, represent, and interpret data on two categorical and quantitative variables.

Math Clusters for Liner, Quadratic, and Exponential Models: Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation they model.

Montana Common Core Standards for Mathematics and Mathematical Practice
http://montanateach.org/wp-content/uploads/2017/10/K-12-Montana-Math-Standards.pdf

Math Standards for Statistics and Probability – Interpreting Categorical and Quantitative Data:

S-ID.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

Math Standards for Statistics and Probability – Making Inferences and Justifying Conclusions:

S-IC.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Math Standards for Functions – Linear, Quadratic, and Exponential Models:

F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.

Indian Education for All Essential Understandings

EU 4: Reservations are lands that have been reserved by the tribes for their own use through treaties, statutes, and executive orders and were not "given" to them.

Learning Objectives

In this lesson, students will . . .

- create a scatter plot, using appropriate technology, to represent the population data of the American Indians and total population for all Montana reservations and their related areas;
- use their model to make predictions of Montana's American Indian populations;
- use residual plots to decide if a model is a good fit;
- find a model that best fits the data either by hand or using appropriate technology;
- make predictions using their model;
- create a residual plot for a data set.

Assessment

Formative Assessment:

- Participation with group discussion
- Observation of students' graphs and method of finding their model to fit the data
- Individual questioning of students

Summative Assessment:

- Graphs and models for data sets of American Indian and total population for Montana reservations and their related areas
- Worksheet with questions answered and turned in
- Review Essential Questions:
    - What changes have occurred in the American Indian population on Montana reservations and related areas over the years 2000-2006?
    - What changes have occurred in the total population on Montana reservations and related areas over the years 2000-2006?
    - How do the changes compare for the two different populations?
    - What factors seem to account for the changes between American Indian reservation populations and the total populations of the reservation and related areas?
    - What factors seem to account for the changes between reservation populations?
    - Is there more than one type of model that could fit a data set?
    - How do we decide if a model is a reasonable model for a data set?

Materials and Resources

For students:

- Copies of the corresponding Demographic and Economic Information pamphlet (Montana Department of Labor and Industry) for each Montana reservation for each student in each assigned reservation group.
- Copies of the student worksheet "Montana Reservation Populations."

- Graph paper or large paper to create a scatter plot on. Scatter plots could also be created on a type of technology that can be shown to or used by all students.
- Graphing utility or device capable of calculating the best fit model.

For teachers:

- There is additional information for the teacher attached to this lesson that provides a summary of the data found in the Demographic and Economic Information pamphlets for March, 2008 for the seven Montana reservations. This is the information that needs to be handed out to students.
- There are two data charts at the end of this lesson providing information that can be used as part of an extension lesson. These data charts (one for the students and one for the teacher) show the Population Percentage by Age Group and Reservation.

Teacher Preparation

Prior to class, review the information provided in the Demographic and Economic Information pamphlets to understand the way the data is presented. The total American Indian population and Total Population on Montana reservations and related areas can be calculated by adding the county populations where the reservations reside. For example, the total American Indian population for the Fort Peck Reservation/Related Areas is the sum of the American Indian populations for Daniels, Roosevelt, Sheridan, and Valley counties.

Instructional Plan

Learning Activity 1

1. Review the vocabulary for this lesson:
   Reservation (See EU 4)
   Scatter plot
   Linear
   Exponential
   Residual
   Trend

2. Introductory activity: Divide the class into seven groups, representing the seven Montana reservations that will be analyzed. Hand out Demographic and Economic Information pamphlets for each Montana reservation to the group representing that reservation. Each student in the group receives one for the reservation they have been assigned. After students have had a few minutes to review their information, you may need to explain how the Total Population means as explained in the Teacher Preparation notes above (The total American Indian population and total population on Montana reservations and related areas can be calculated by adding the county populations where the reservation resides. For example, the total American Indian population for the Fort Peck Reservation/Related Areas is the sum of the American Indian populations for Daniels, Roosevelt, Sheridan and Valley counties.). Ask the class the following questions:

- How do American Indian and Total Population change? Why do you think this is the case? Do you believe they are interrelated and why?
- Does the Total Population for the same reported counties tend to increase or decrease?

These are intended to be thought-provoking questions to see the students' initial responses and there will not be absolute responses.

3. Have the students complete questions 1- 4 on the student worksheet for their reservation in their group.

4. Have each group post their scatter plots around the room so they are visible for the entire class.

5. Ask the students to examine the scatter plots for each reservation (possibly using a Gallery Walk), and then have the groups discuss the following questions:
   - **What type of model(s) best represents the data change?**
   - With what we know about populations, what type of model best fits this trend?

   Have groups share back their initial thoughts with the class as to what they think the most appropriate model(s) would be.

6. Have the students complete questions 5 - 6 on the student worksheet in their reservation group.

7. Have each group write their model of best fit alongside their posted corresponding scatter plot.
   - What is common about the given models?
   - What is different?

8. After a discussion around questions 5 and 6, have the students complete questions 7-11 in their reservation groups. This may require some reminder discussions about residuals and residual plots. This may also be done as an assignment with the students sharing answers the next class within their groups and coming to consensus on the answers. When the reservation groups have completed these questions, have the groups share responses or have the worksheet turned in as an assignment.

9. Use the essential questions listed in the Assessment section above as a closing activity or as a summative assessment.

Extension Activities and Additional Resources

- More current information may be found for each of the Montana reservations at http://lmi.mt.gov/Publications/PublicationsContainer/PID/34826/evl/0/CategoryID/307/CategoryName/Reservation-Profiles. These could be used to update the data analysis.

- An extension lesson could focus on the population percentage of American Indians by age group and the population percentage of Montanans by age group. Data for 2000-2006 is attached at the end of this lesson. That data can be modeled by a logistic function for both each tribe compared to the Montanan percentage. Discussing with students why the same type of model would not work for this data could lead to an interesting discussion.

Student Worksheet
Montana Reservation Populations

Names: _____ **Period: _____**

Reservation: _____

1. Create a scatter plot representing the data for the American Indian Population for the years 2000-2006. Let *x* represent the number of years after 2000: {*x* = 0 represents 2000 and *x* = 1 represent 2001, etc.}

2. Explain the trend you see that exists with your data sets.

3. Create a scatter plot representing the data for the total population for the years 2000-2006. Let x represent the number of years after 2000: {x = 0 represents 2000 and x = 1 represent 2001, etc.}

4. Does the same trend exist for the total population as did in the American Indian population? Compare and contrast the trends.

5. What possible model(s) would best fit your population? Determine the model for each of your data sets. Be sure to write your model in the correct form.

6. Determine the predicted population values for the years 2000-2006 using the models found in question 5. Use the predicted values to calculate the residuals for your data sets.

7. What does a negative residual value mean? What does a positive residual value mean?

8. Create a residual plot for your assigned American Indian Population and a separate residual plot for the Total Population for your reservation and related areas. Do the residual plots indicate that your models are a reasonable model?

9. Use your model(s) to predict the population(s) of the American Indian and total for **2007**. Should we use our model to predict for the year **2012**?

10. Create a residual plot for a different type of model that "may" fit the data sets. Which model is the best fit after looking at your different residual plots?

11. After looking at the population trends of your models, what can you conclude about the future of the two populations? What are some external factors that may be the cause of the population trends you saw?

Student Worksheet (ANSWER KEY)
Montana Reservation Population

1. Create a scatter plot representing the data for the American Indian Population for the years 2000-2006.

   Scatter plots vary depending upon the reservation of focus. It is important that the plots are easily read by the entire class. The scatter plot could be created on a graphing calculator and then displayed on an overhead view screen or Smartboard if available.

2. What trend do you see exists with your data set?

   Generally speaking the tribal populations are increasing. Some of the total populations are increasing and others are decreasing. See the data chart, Additional Information for Teachers, provided at the end of this lesson for a summary of the percentage changes.

3. Create a scatter plot representing the data for the total population for the years 2000-2006.

4. Does the same trend exist for the total population as did in the American Indian population? Compare and contrast the trends.

   This response will be dependent upon their specific reservation of focus.

5. What possible model(s) would best fit your populations? Determine the model for each of your data sets. Be sure to write your model in the correct form.

   Some students will believe that the data looks very linear since it is for such a small amount of time. Others will believe that it represents an exponential model since we have an initial population and then percentage of change. Student should experiment with several models to determine which one best fits their particular populations. See the possible exponential models for each reservation and related areas in the chart below.

6. Determine the predicted population values for the years 2000-2006 using the models found in question 5. Use the predicted values to calculate the residuals for your data sets.

   Answers will vary based upon their specific reservation.

7. What does a negative residual value mean? What does a positive residual value mean?

   A negative residual means the predicted value is larger than the actual value. A positive residual value means the predicted value is smaller than the actual value.

8. Create a residual plot for your assigned American Indian Population and a separate residual plot for the Total Population for your reservation and related areas. Do the residual plots indicate that your models are reasonable?

   If the residual plot (x-value, residual) has the points randomly scattered above and below the x-axis, this graph indicates that a reasonable model has been selected.

9. Use your model(s) to predict the population(s) of the American Indian and total for **2007**. Should we use our model to predict for the year **2012**?

   These values will depend upon the model used. Predicting outside of our data set is not a good practice, especially with an exponential model. There is data from the 2010 census that can be used. It is found for for each of the Montana reservations on individual brochures available at: http://lmi.mt.gov/Publications/PublicationsContainer/PID/34826/evl/0/CategoryID/307/CategoryName/Reservation-Profiles.

10. Create a residual plot for a different type of model that "may" fit the data sets. Which model is the best fit after looking at your different residual plots?

    *As stated in question 5, some students will try a linear model here**.** Students should be encouraged to try an exponential model.*

11. After looking at the population trends of your models, what can you conclude about the future of both the American Indian Population and the Total Population for your assigned reservation and related areas? What are some external factors that may be the cause of the population trends you saw?

    Answers may vary. Students may consider things like age (young leave for college, work, etc.), employment opportunities, geographic location, natural resources, proximity to population centers, weather patterns (that may affect agriculture), etc.

The table below represents the possible exponential models for the American Indian Population by reservation and for the total population using the data provided on the 2008 pamphlets.

Reservation (American Indian Population)	**Exponential Model**
Fort Peck	y = 6657.5 * 1.008^x
Fort Belknap	y = 3444.5 * 1.0009^x
Blackfeet	y = 8928.8 1.007^x
Northern Cheyenne	y = 10523.7 * 1.009^x
Rocky Boy's	y = 3738.1 * 1.01^x
Flathead	y = 9972.8 * 1.013^x
Crow	y = 11574.3 * 1.02^x
Reservation (Total Population)	**Exponential Model**
Fort Peck	y = 24278 * 0.99^x
Fort Belknap	y = 11490 * 0.987^x
Blackfeet	y = 19533.7 * 1.0004^x
Northern Cheyenne	y = 22042 * 1.002^x
Rocky Boy's	y = 22426.5 * 0.99^x
Flathead	y = 208630 * 1.012^x
Crow	y = 141820 * 1.011^x

Additional Information for Teachers

Here is summary information for the teacher regarding the Total Population and American Indian population by related areas based on the Demographic & Economic Information for the reservations (2008) provided to the students. The percentage change of both the American Indian population and total population is indicated along with the average percentage of change by Reservation.

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Fort Peck Related Areas: Counties of Daniels, Roosevelt, Sheridan, and Valley	2000	6679	1.32%	24417	-1.45%
	2001	6767	-0.93%	24064	-2.09%
	2002	6704	0.81%	23562	-1.25%
	2003	6758	1.97%	23267	0.25%
	2004	6891	1.86%	23326	-1.02%
	2005	7019	-0.44%	23087	-1.62%
	2006	6988	**(average)** 0.76%	22712	**(average)** -1.20%

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Fort Belknap Related Areas: Counties of Blaine and Phillips	2000	3502	-2.28%	11610	-2.93%
	2001	3422	0.15%	11270	-0.91%
	2002	3427	-0.03%	11167	-1.32%
	2003	3426	0.18%	11020	-1.23%
	2004	3432	1.19%	10884	-1.06%
	2005	3473	0.63%	10769	-0.52%
	2006	3495	**(average)** -0.03%	10713	**(average)** -1.33%

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Northern Cheyenne and Related Areas: Counties of Big Horn and Rosebud	2000	10539	0.98%	22054	0.16%
	2001	10642	0.38%	22090	0.06%
	2002	10682	0.64%	22104	0.37%
	2003	10750	1.69%	22186	0.44%
	2004	10932	1.49%	22284	0.32%
	2005	11095	-0.23%	22355	-0.26%
	2006	11069	**(average)** 0.82%	22296	**(average)** 0.18%

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Rocky Boy's and Related Areas: Counties of Choteau and Hill	2000	3736	1.98%	22643	-1.52%
	2001	3810	-0.45%	22299	-1.37%
	2002	3793	1.11%	21994	-0.70%
	2003	3835	1.36%	21840	0.17%
	2004	3887	1.21%	21877	-0.63%
	2005	3934	1.40%	21740	0.37%
	2006	3989	**(average)** 1.10%	21820	**(average)** -0.61%

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Flathead and Related Areas: Counties of Flathead, Lake, Missoula, Sanders	2000	9970	1.46%	211048	-0.40%
	2001	10116	0.90%	210200	1.17%
	2002	10207	1.43%	212662	1.38%
	2003	10353	1.31%	215600	1.52%
	2004	10489	0.95%	218879	1.61%
	2005	10589	1.65%	222396	1.83%
	2006	10764	**(average)** 1.29%	226475	**(average)** 1.19%

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Blackfeet and Related Areas: Counties of Glacier and Pondera	2000	8990	-0.50%	19671	-0.95%
	2001	8945	0.84%	19485	-0.18%
	2002	9020	0.62%	19450	0.10%
	2003	9076	1.18%	19469	0.75%
	2004	9183	0.73%	19615	-0.05%
	2005	9250	0.49%	19606	0.02%
	2006	9295	**(average)** 0.56%	19610	**(average)** -0.05%

Reservation	Year	American Indian Pop.	Percentage Change	Total Population	Percentage Change
Crow and Related Areas: Counties of Big Horn and Yellowstone	2000	11630	1.87%	142023	0.97%
	2001	11847	1.93%	143399	0.92%
	2002	12076	0.95%	144715	0.92%
	2003	12191	2.39%	146044	1.20%
	2004	12482	2.85%	147796	1.26%
	2005	12838	3.05%	149662	1.06%
	2006	13229	**(average)** 2.17%	151248	**(average)** 1.05%

Data for Possible Extension Activity
Student Information: Percentages used to create the Percentage Pyramids

Population Percentage by Age Group and Reservation: The Little Shell Chippewa Tribe and the Blackfeet Tribe do have percentages by age group also, but is not included in this information.

Age Group	Montana Percent	Crow Reservation	Flathead Reservation	Fort Belknap Reservation	Fort Peck Reservation	Northern Cheyenne Reservation	Rocky Boy Reservation
0-4	6.1%	10.0%	7.1%	10.1%	8.6%	11.5%	12.0%
5-9	6.9%	10.2%	7.6%	11.1%	10.2%	12.1%	12.5%
10-14	7.7%	10.7%	8.8%	12.9%	11.1%	12.8%	12.5%
15-19	7.9%	10.3%	8.6%	11.3%	9.1%	10.6%	10.6%
20-24	6.5%	6.4%	5.2%	6.1%	5.5%	6.1%	7.7%
25-29	5.7%	5.3%	4.9%	4.9%	5.7%	6.6%	5.7%
30-34	5.8%	6.1%	5.4%	5.6%	5.4%	6.7%	6.2%
35-39	7.4%	7.6%	6.9%	7.6%	7.1%	6.6%	7.3%
40-44	8.4%	7.3%	8.0%	7.4%	8.3%	6.1%	6.4%
45-49	8.1%	6.3%	7.7%	5.8%	7.1%	5.2%	5.2%
50-54	6.8%	5.8%	6.6%	4.4%	5.4%	5.6%	3.6%
55-59	5.2%	4.2%	5.4%	3.3%	3.9%	3.4%	3.3%
60-64	4.2%	3.6%	4.6%	3.0%	3.2%	2.7%	2.2%
65-69	3.6%	2.4%	4.0%	2.4%	3.0%	1.8%	2.4%
70-74	3.3%	1.6%	3.7%	1.9%	2.5%	1.1%	1.1%
75-79	2.7%	1.3%	2.9%	1.3%	1.9%	0.9%	0.8%
80-84	2.0%	0.7%	2.0%	0.6%	1.6%	0.3%	0.5%
85+	1.7%	0.5%	1.9%	0.4%	1.3%	0.4%	0.2%

Data for Possible Extension Activity
Teacher Information: Percentages used to create the Percentage Pyramids

This information is the difference between the total percentage of Montanans and the following tribes. A negative value indicates that the tribal percentage in that age group was less than that of all Montanans. The standard deviation value represents the standard deviation for that age group.

Age Group	Crow Difference	Flathead Difference	Fort Belknap Difference	Fort Peck Difference	Northern Cheyenne Difference	Rocky Boy Difference	Standard Deviation
0-4	3.9%	1.0%	4.0%	2.5%	5.4%	5.9%	0.01659851
5-9	3.4%	0.8%	4.2%	3.3%	5.2%	5.7%	0.01603296
10-14	3.1%	1.1%	5.3%	3.4%	5.1%	4.8%	0.01448941
15-19	2.4%	0.7%	3.4%	1.2%	2.7%	2.7%	0.00939335
20-24	-0.1%	-1.2%	-0.3%	-1.0%	-0.4%	1.2%	0.00783976
25-29	-0.3%	-0.8%	-0.8%	0.1%	0.9%	0.0%	0.00572292
30-34	0.3%	-0.4%	-0.2%	-0.4%	0.9%	0.4%	0.00483038
35-39	0.2%	-0.5%	0.2%	-0.3%	-0.8%	-0.1%	0.00375261
40-44	-1.1%	-0.3%	-1.0%	-0.1%	-2.3%	-1.9%	0.00793263
45-49	-1.8%	-0.4%	-2.3%	-1.0%	-2.9%	-3.0%	0.00935149
50-54	-1.0%	-0.2%	-2.4%	-1.4%	-1.3%	-3.2%	0.00964459
55-59	-1.0%	0.2%	-1.9%	-1.3%	-1.8%	-1.9%	0.00744209
60-64	-0.6%	0.4%	-1.2%	-1.0%	-1.5%	-2.0%	0.00748328
65-69	-1.2%	0.4%	-1.2%	-0.6%	-1.8%	-1.2%	0.00697562
70-74	-1.7%	0.4%	-1.4%	-0.8%	-2.2%	-2.2%	0.00917058
75-79	-1.4%	0.1%	-1.5%	-0.9%	-1.8%	-1.9%	0.00691851
80-84	-1.4%	-0.1%	-1.5%	-0.4%	-1.7%	-1.5%	0.00615795
85+	-1.2%	0.2%	-1.3%	-0.4%	-1.3%	-1.5%	0.00601474